Significance Level Worksheet Name:_____

The Bob Barker Foundation, in collaboration with the Stud Dog Union, report that 58% of male dogs are neutered. You believe that the proportion is actually larger. You decide to taking a random sample of 100 dogs to gather more information. We will use $\alpha = 0.05$.

Questions

0. The null and alternative hypotheses are:

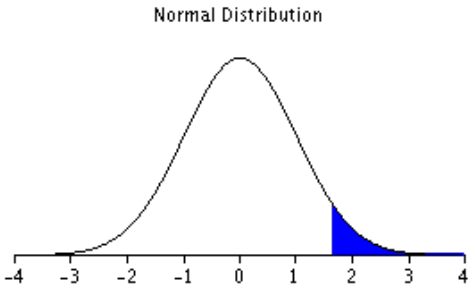
$$H_0 : p = 0.58 \qquad H_A : p > 0.58$$

1. Assuming H_0 is true and all conditions are satisfied (they are), the sampling distribution of $\hat{p}$ for random samples of size $n = 100$ is:

$$\hat{p} \sim \text{N}(\ 0.58,\ 0.049)$$

2. Using the sampling distribution assumed under the null hypothesis, find the value of $\hat{p}$ for which the top 5% of all $\hat{p}$'s will be above. *hint:* $z = 1.645$

$$\hat{p} \quad = \quad z \times (SD_{\hat{p}}) + p$$



The shaded region above is called the "rejection region". If the $\hat{p}$ we observe from our sample is in this region we will "Reject the Null Hypothesis". If it is not in this region we will "Fail to Reject the Null Hypothesis".

3. We take a random sample of size $n = 100$ and find that 68 out of 100 dogs were neutered. Is this sample proportion in the "rejection region"? Explain.

4. Calculate the p-value for the sample $\hat{p}$ from question 3.

5. Comparing this p-value to the significance level of $\alpha = 0.05$, what is your decision about the Null Hypothesis? Explain.

6. Assume your intuition is correct and the true proportion of dogs that are neutered is 0.74. What proportion of $\hat{p}$'s do you expect to be greater than the cutoff from question 2. That is, what proportion of sample $\hat{p}$ values do you expect will be in the rejection region. *hint: the sampling distribution for $\hat{p}$ is actually*

$$\hat{p} \sim \text{N}(\ 0.74,\ 0.044)$$

7. Give the probabilities of the following four cases:

   (a) **Type 1 Error**: H_0 is true and we reject H_0: $\alpha =$

   (b) H_0 is true and we fail to reject H_0: $1 - \alpha =$

8. The following 2 cases are assuming the scenario in Question 6 is the truth.

   (c) **Type 2 Error**: H_0 is false and we fail to reject H_0: $\beta =$ *1 - answer from Q.6:*

   (d) **Power**: H_0 is false and we reject H_0: *answer from Q.6:*