

## CHAPTER 19 NOTES

### CONFIDENCE INTERVALS FOR PROPORTIONS

#### KING PENGUIN STUDY

Flipper bands have been used for decades to identify individual penguins so they can be tracked on land and sea. The French researchers randomly selected 50 king penguins from a colony on Possession Island off the Antarctic coast, and tagged them with flipper bands. After a decade of observation only 10 were still alive.



1

#### Inference on $p$

- In chapter 18I, we found that the long-term behavior of the sample proportion,  $\hat{p}$ , can be described using a normal distribution (under certain conditions)

$$N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

- Since we know the distribution of  $\hat{p}$ , we know how to describe the uncertainty.

#### Standard Error

- We can replace  $p$  with  $\hat{p}$  in the formula for standard deviation.

3

#### Inference on $p$

We wish to estimate the population proportion,  $p$ , using the sample proportion,  $\hat{p}$ . So far, we have only discussed using the sample statistic as a point estimate of the parameter, however, this estimate ignores the uncertainty associated with the sampling process.

Recall: due to sampling variability

2

#### Review of the 68-95-99.7 Rule

Recall:

- approx. 68% of all  $\hat{p}$  values fall within 1 standard error of  $p$
- approx. 95% of all  $\hat{p}$  values fall within 2 standard error of  $p$
- approx. 99.7% of all  $\hat{p}$  values fall within 3 standard error of  $p$

Consider the 95% level:

4

### Confidence Interval for $p$

The general form of a confidence interval is:

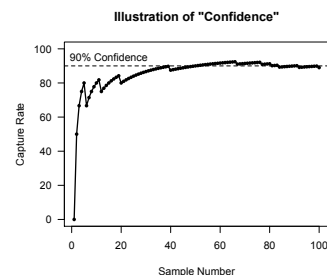
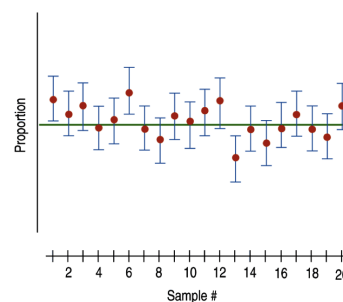
$$\text{estimate} \pm ds_n \times SE$$

So, a confidence interval for a population proportion is given by

### What Does *Confidence* Really Mean?

- A sample statistic is a random quantity, and each confidence interval is constructed *around* a sample statistic, so the confidence intervals themselves are in fact **random**.

### What Does *Confidence* Really Mean?



### Practice Finding $z^*$ Values

1. What is  $z^*$  for the 90% confidence level?



### Practice Finding $z^*$ Values

4. What is  $z^*$  for the 98% confidence level?



2. What is  $z^*$  for the 95% confidence level?



5. What is  $z^*$  for the 99% confidence level?



#### Example 1

In 2004, ACT, Inc. reported that 74% of 1644 randomly selected college freshman returned to college the next year. Find a 98% confidence interval for the national freshman-to-sophomore retention rate.

Step 1 Identify the important information in the problem.

Step 2 Check the assumptions/conditions needed to use a normal sampling distribution.

Step 3 Find the appropriate  $z^*$  from the z-table.

Step 4 Calculate the confidence interval.

Step 5 Provide an interpretation for the confidence interval.

#### Example 2

In a May 2007 Experian/Gallup Personal Credit Index poll of 1008 randomly selected U.S. adults aged 18 and over, 8% of respondents said they were very uncomfortable with their ability to make their monthly payments on their current debt during the next three months. Find a 95% confidence interval for the proportion of the adult population that feels this way.

Step 1 Identify the important information in the problem.

Step 2 Check the assumptions/conditions needed to use a normal sampling distribution.

Step 3 Find the appropriate  $z^*$  from the z-table.

Step 4 Calculate the confidence interval.

Step 5 Provide an interpretation for the confidence interval.

### Example 3

A random sample of 539 households in Minneapolis was selected, and it was determined that 133 of these households owned at least one firearm (“The Social Determinants of Gun Ownership: Self Protection in an Urban Environment,” *Criminology*, 1997: 629–640). Find a 90% confidence interval for the proportion of all households in Minneapolis that own at least one firearm.

Step 1 Identify the important information in the problem.

Step 2 Check the assumptions/conditions needed to use a normal sampling distribution.

Step 3 Find the appropriate  $z^*$  from the z-table.

Step 4 Calculate the confidence interval.

Step 5 Provide an interpretation for the confidence interval.

### Properties of Confidence Intervals

- **Margin of Error**

- **Width**

### Properties of Margin of Error

What is the effect of confidence level on the margin of error?

### Properties of Margin of Error

What is the effect of sample size on the margin of error?

### The Goal of Confidence Intervals

When we construct confidence intervals we want to have both

### Finding the Sample Size

Before taking a sample, we can determine what sample size we should use for a specified confidence level and margin of error.

### Example 4

In preparing a report on the economy, we need to estimate the percentage of businesses that plan to hire additional employees in the next 60 days. How many randomly selected employers must we contact in order to create an estimate in which we are 98% confident with a margin of error of 5%?

### Finding the Sample Size

**PROBLEM:** We do not know the value of  $p$ , so what do we do now?

### Example 5

A state's environmental agency worries that many cars may be violating clean air emissions standards. The agency hopes to check a sample of vehicles in order to estimate that percentage with a margin of error of 3% and 90% confidence. To gauge the size of the problem, the agency first picks 60 cars and finds 9 with faulty emissions systems. How many cars should be sampled for the full investigation?